MAKING MATHEMATICS DELICIOUS

SOLVING EIGHTH-GRADE MATH PROBLEMS
IN THE KITCHEN & IN THE GARDEN

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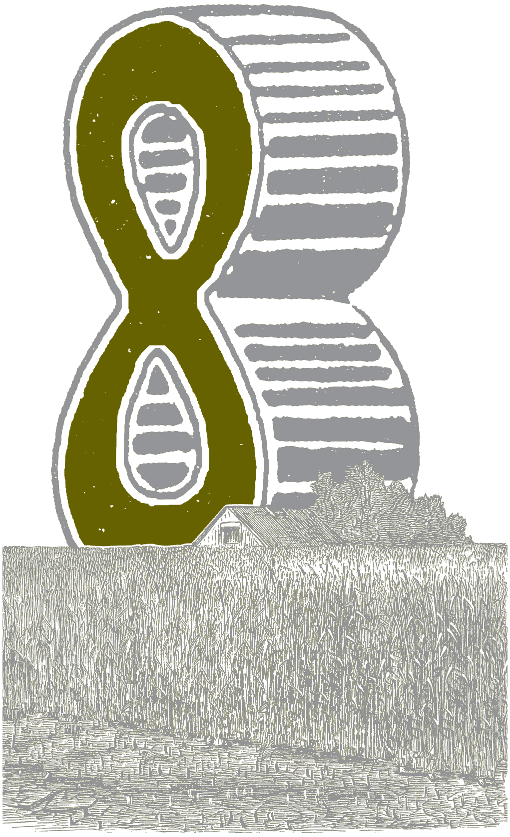


CHEZ PANISSE FOUNDATION

Cultivating a New Generation

FOREWORD

Our mission at the Chez Panisse Foundation is to educate, nurture, and empower students to build a more sustainable future. Our program, the Edible Schoolyard, brings this mission to life by involving students in growing, harvesting, cooking and enjoying simply-cooked, seasonal, organic meals. We lure students into the kitchen with delicious smells and flavors and find them excited by the simple pleasures of the garden—picking raspberries, digging up beds, or planting seeds.

"Learning by doing" is central to the way we teach at the Edible Schoolyard. In our kitchen and garden classes, students participate fully in the cycle of producing organic and seasonal foods. They may bring kitchen and garden waste to the compost pile, for example, turn and tend the pile, spread the finished compost on garden beds before planting lettuce or tomatoes, then use the harvest from these plants in a delicious salad for sharing.

We also strive to bring the academic classroom to life, whether it's through a social studies lesson on ancient grains or a science lesson on the composition and properties of soil. For the last two years we have also been exploring how to weave meaningful mathematics into our instruction. We do this because all students need mathematics skills to become socially responsible leaders and productive citizens and, in the short term, to enter college without need for remediation.

This book is not a comprehensive math curriculum. Rather, it offers a window onto our approach to teaching and learning. Teachers may adapt these lessons according to the resources and concerns of their own classrooms, or simply use them for inspiration. Our goal is to empower students to grapple with the intrinsic complexities of mathematics in a way that is purposeful and not devoid of common sense; with this book, we hope to provide a starting place.

Carina Wong
Executive Director
Chez Panisse Foundation

When we began thinking about the importance of integrating kitchen and garden learning into the academic classroom, we did not have a mathematics book in mind. Soon, however, teachers at the Edible Schoolyard began developing their own strategies for engaging students with mathematics. *Making Mathematics Delicious* is one product of the collective wisdom we have developed.

This book differs from many supplementary curriculum materials in its alignment with California mathematics standards for middle school. This table lists the mathematics content standards that inform our lessons:

Grade	Content Strand Mathematics Content	Standard
8	Algebra & Functions	8.6.0
	Students graph a linear equation and compute the x- and y- intercepts (e.g., graph $2x + 6y = 4$). They are also able to sketch the region defined by linear inequality (e.g., they sketch the region defined by $2x + 6y < 4$).	
8	Algebra & Functions	8.7.0
	Students verify that a point lies on a line, given an equation of the line. Students are able to derive linear equations using the point-slope formula.	
8	Algebra & Functions	8.10.0
	Students add, subtract, multiply, and divide monomials and polynomials. Students solve multi-step problems, including word problems, by using these techniques.	
8	Algebra & Functions	8.15.0
	Students apply algebraic techniques to solve rate problems, work problems, and percent mixture problems.	

The context for each assignment is drawn directly from the original Edible Schoolyard at Martin Luther King Jr. Middle School in Berkeley, California. However, middle school teachers from all regions will find these assignments relevant and useful for any mathematics classroom, with or without an adjoining school garden.

These mathematics assignments are what we call *tasks*. Tasks are problem-based and set in a real-world context. They involve multiple steps and usually cannot be completed in one class period. Nor can the tasks simply be assigned to the students as handouts. The tasks require conversation, experimentation, and at times collaboration. You may also notice that the tasks involve a lot of reading; this is because a task must require sense-making on the part of the student if it is to be a truly useful tool for learning. Each task is also designed to be accessible for all learners and to promote mathematical thinking over imitative computation work.

When students make bread dough in the kitchen, for example, they must estimate to decide when it has doubled in volume. Thus making bread provides an excellent opportunity for students to understand the relationship between concomitant increases in volume and linear dimensions. In the garden, students make potting soils to nurture plants through various stages of growth. The recipe for each type of soil is presented in a variety of standard and non-standard units because in the real world, gardeners often use cans, wheelbarrows, and spades to measure. This experience allows students to hone their quantitative literacy while also deepening their understanding of how gardeners work.

HOW THIS BOOK IS ORGANIZED

This book includes five tasks for use with eighth-grade students. Each task is introduced using handmade recipes, drawings, or images from the Edible Schoolyard. For the teacher, a set of notes is provided to explain underlying mathematical concepts and how to approach the lesson. The solution to each task is also included. Related tasks are outlined in booklets for seventh and eighth grades.

We published the three grades as a set so you could see how similar recipes or contexts can provide tasks of increasing difficulty. For example, the sixth-grade lesson using a structure in the Edible Schoolyard called the Ramada involves measuring angles and calculating area. At the eighth-grade level, the Ramada task involves constructing a scale model using measurements from the actual structure at the Edible Schoolyard.

We hope that these tasks will inspire you to look around and find the math in your kitchen, garden, or community.

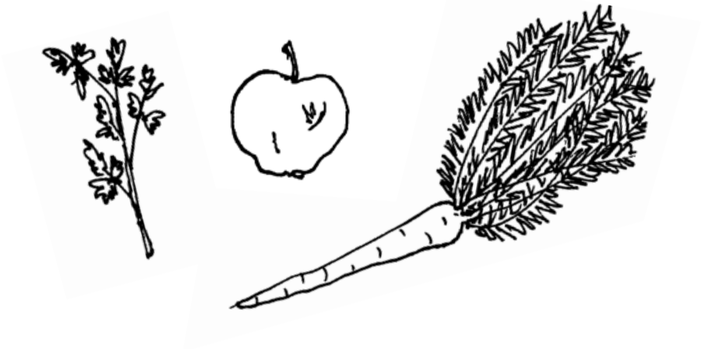
MAKING MATHEMATICS DELICIOUS • GRADE 8

PROBLEMS & QUESTIONS FOR STUDENTS

TEACHER NOTES & SOLUTIONS

THE MATHEMATICS OF APPLE AND CARROT SALAD WITH SESAME OIL DRESSING

Apple and Carrot Salad with sesame oil dressing



- Wash apples and cilantro
- Wash and peel carrots
- Cut apples and carrots in to match sticks
- pick cilantro leaves off the stem – roughly chop the cilantro leaves

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- dressing:

  2 tablespoons rice vinegar

  1 tablespoon sesame oil

  1/4 teaspoon salt
- Pour dressing over salad – toss to mix and serve.

Exploring a Recipe

1. *What percentage of liquid ingredients in the sesame dressing is oil?*

2. *The cooks at one school want to increase the proportion of olive oil in their school lunch salad dressing because olive oil is so good for students' health.*

   The dressing they use now contains 25% olive oil. How much olive oil must be added to 6 liters of that dressing to make a new dressing containing 50% olive oil? Make a diagram to help you.

THE MATHEMATICS OF WHOLE-WHEAT FRENCH DOUGH

WHOLE-WHEAT FRENCH DOUGH

INGREDIENTS:

2 CUPS WARM WATER
1 T SUGAR
1½ T YEAST
1 T SALT
1 CUP WHOLE-WHEAT FLOUR
4 CUPS WHITE FLOUR
OLIVE OIL

EQUIPMENT:

1 MEDIUM MIXING BOWL
1 SET MEASURING SPOON
1 MEASURING CUP
1 WOODEN SPOON
1 CUTTING BOARD

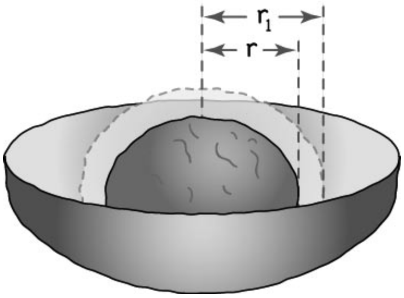
Put warm water in the mixing bowl. Sprinkle in sugar and yeast, stir gently to mix and watch the yeast COME TO LIFE! Wait until it gets nice and foamy - about 5 minutes. Add 1 cup whole-wheat flour + 1 cup white flour and 1 T salt, stirring until well-mixed. Slowly add in 2 more cups of white flour while mixing. Turn dough out onto a lightly floured cutting board and knead until smooth. Add in the remaining cup of flour a needed. Do not add more than 5 cups total or the dough will become tough and dry.

Wash and dry the mixing bowl and grease the inside lightly with oil. Place dough in the bowl and cover with plastic wrap. Place dough in a warm place to rise. This takes about 1 hour– until the dough has doubled in size. Now it is ready to form into loaves or rolls.

Measuring a Ball of Dough

The recipe for Whole-Wheat French Dough says that the dough should be left to rise until doubled. This means that the dough is left until its volume has increased by a factor of 2.

The diagram below shows a sketch of a before-rising ball of dough with radius r and a sketch of an after-rising ball of dough with radius r_1.



You know that the after-rising volume will be twice the before-rising volume. What will be the relationship between the before- and after-rising radii of the ball of dough?

1. *When the before-rising radius of a ball of dough (sphere) is* r, *write an expression for the volume of a ball of dough.*

2. *Write an expression for the volume of a ball of dough when the after-rising radius is* r_1.

3. *When the before-rising radius of a ball of dough is* r *and the after- rising radius of the dough is* r_1, *then the following is true:*

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi r_1^3$$

   Solve this equation for r_1 *to find an expression for the after-rising radius of the ball of dough.*

4. *When the dough doubles in volume, by what factor does* r_1, *the after-rising radius, increase?*

5. *Complete the following table.*

When a Ball of Dough Doubles in Volume

before-rising radius r *in centimeters*	*after-rising radius* r_1 *in centimeters*
4	
5	
6	
7	

6. *Create a formula to express the after-rising radius in terms of the before-rising radius.*

7. *Create a graph showing* r *on the horizontal axis and* r_1 *on the vertical axis.*

8. *What is the slope of the line?*

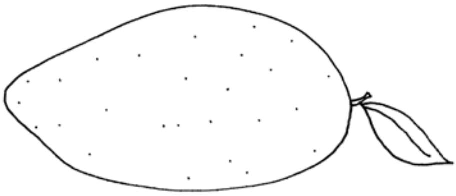
9. *What does the slope mean in terms of the rising dough situation?*

Lemonade

juice of six lemons

½ cup sugar

1 pitcher water



Increasing a Recipe

The recipe for Lemonade tells us that for the juice of every 6 lemons, ½ cup of sugar is needed.

1. *Complete this table to show the number of lemons and the corresponding amounts of sugar required when the recipe is increased.*

Quantities of Lemons and Sugar Needed for Lemonade

lemons	*cups of sugar*
3	
6	½
9	
12	

2. *We can think of the number of lemons and the number of cups of sugar as two variables.*

   Let X *represent the number of lemons.*
   Let Y *represent the number of cups of sugar.*

   a. *Which of these two variables does it make sense to call the* independent variable*? Explain why.*

   b. *Which of these two variables does it make sense to call the* dependent variable*? Explain why.*

c. *Is the relationship between the number of lemons and the number of cups of sugar a* function? *Explain.*

3. *Write a formula to express the number of cups of sugar in the recipe in terms of the number of lemons.*

4. *Draw a graph to show the relationship between the number of lemons and the number of cups of sugar required when the recipe is increased. Record the number of lemons along the* x-*axis and the number of cups of sugar along the* y-*axis.*

5. *What is the ratio of the number of cups of sugar to the number of lemons?*

6. *What is the slope of the line?*

7. *What does the slope represent in terms of making lemonade?*

8. *Does the line pass through the point (0,0)?*

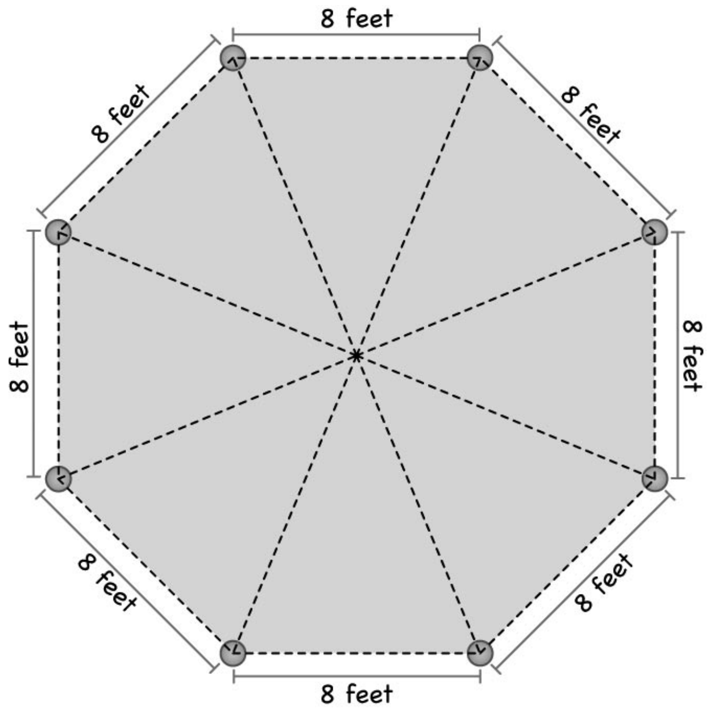
9. *Does the number of cups of sugar needed to make lemonade vary directly with the number of lemons used? Explain how you know.*

The Ramada

Measuring the Ramada

The Ramada is a shade structure and gathering place in the Edible Schoolyard.

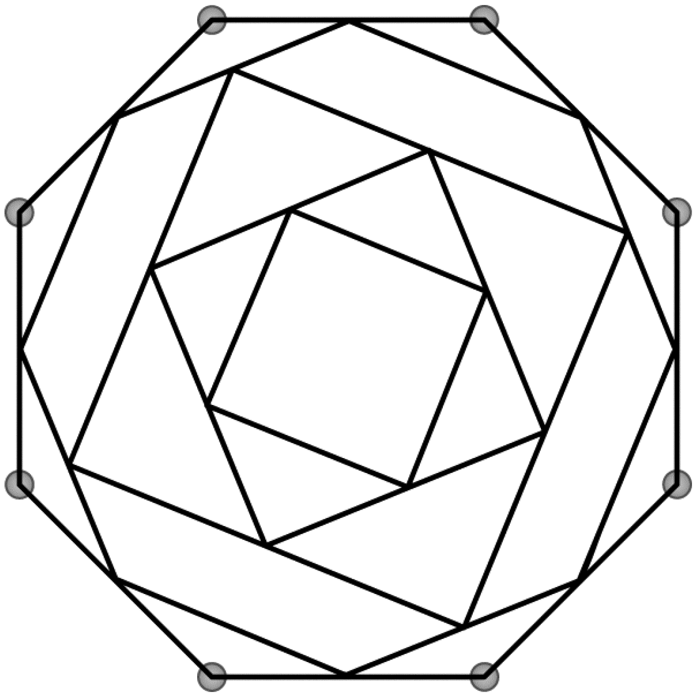
Here is a drawing of the floor plan of the Ramada.



The perpendicular distance from any one "wall" to the center of the Ramada is 9.7 feet.

Bales of straw, each 4 feet by 2 feet by 1 foot, line 7 walls of the Ramada.

Here is a drawing of the ceiling of the Ramada.



The ceiling of the Ramada rests upon 8 large tree limbs. Each of these stands vertically and is 8 feet high and approximately 8 inches in diameter.

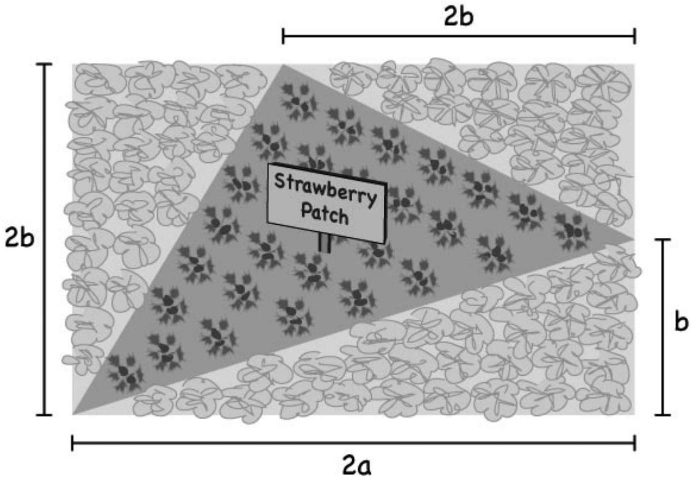
The ceiling itself is constructed using additional tree limbs of various diameters.

1. *Work with classmates to construct a scale model of the Ramada.*

The Variable Garden

Building a Garden Bed

Students at one school are planting a strawberry patch in their school garden. They will plant the patch in a rectangular bed by partitioning the bed into 4 triangles, as shown.



Strawberries will be planted in the triangle in the middle. Ground cover will be planted in each of the 3 right triangles that border the strawberry patch.

1. Work with classmates to create an expression *for the area of the strawberry patch.*

THE MATHEMATICS OF APPLE AND CARROT SALAD WITH SESAME OIL DRESSING—NOTES AND SOLUTION

Percentages and Concentration

Notes

In this task, students are presented with a classic mixture problem like those required in most Algebra 1 courses. Many students find such classic mixture problems extremely difficult and boring. Here we attempt to make the problem more accessible to students by setting it in the context of a recipe.

Solution

Exploring the Recipe

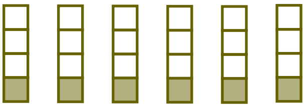
1. *What percentage of liquid ingredients in the sesame dressing is oil?*

   The percentage of liquid ingredients in the sesame dressing that is oil is 33%.

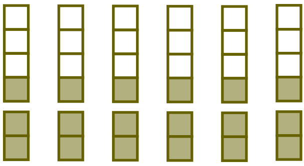
2. *The cooks at one school want to increase the proportion of olive oil in their school lunch salad dressing because olive oil is so good for students' health.*

   The dressing they use now contains 25% olive oil. How much olive oil must be added to 6 liters of that dressing to make a new dressing containing 50% olive oil? Make a diagram to help you.

   The diagram below represents 6 liters of salad dressing. For each liter, 25% is shaded to represent that the dressing contains 25% olive oil.



   The diagram below shows that 0.5 liters of olive oil must be added to each liter of salad dressing to ensure that the salad dressing contains 50% olive oil.



   Therefore, a total of 3 liters of oil must be added to obtain a dressing that is 50% olive oil.

THE MATHEMATICS OF WHOLE-WHEAT FRENCH DOUGH—NOTES AND SOLUTION

Graphing Functions, Analyzing Increases in Volume and Linear Dimensions

Notes

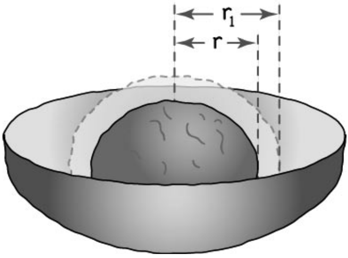
In this task, students use rising bread dough to examine and graph changes in the radius of a sphere. This is a classic mathematics problem. Dough must be left to rise; when the dough has doubled in volume, by how much will the diameter have increased?

Solution

Measuring a Ball of Dough

The recipe for Whole-Wheat French Dough says that the dough should be left to rise until doubled. This means that the dough is left until its volume has increased by a factor of 2.

The diagram below shows a sketch of a before-rising ball of dough with radius r and a sketch of an after-rising ball of dough with radius r_1.



You know that the after-rising volume will be twice the before-rising volume. What will be the relationship between the before- and after-rising radii of the ball of dough?

1. *When the before-rising radius of a ball of dough (sphere) is* r, *write an expression for the volume of a ball of dough.*

   The volume is given by $\frac{4}{3}\pi r^3$.

2. *Write an expression for the volume of a ball of dough when the after-rising radius is* r_1.

   The volume is given by $\frac{4}{3}\pi r_1^3$.

THE MATHEMATICS OF WHOLE-WHEAT FRENCH DOUGH—NOTES AND SOLUTION (CONT.)

3. *When the before-rising radius of a ball of dough is r and the after-rising radius of the dough is* r_1 *then the following is true:*

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi r_1^{\,3}$$

Solve this equation for r_1 *to find an expression for the after-rising radius of the ball of dough.*

An expression for the after-rising radius in terms of r is $\sqrt[3]{2} \bullet r$.

The following shows how it can be derived:

$$\frac{8}{3}\pi r^3 = \frac{4}{3}\pi r_1^{\,3}$$

$$2\pi r^3 = \pi r_1^{\,3}$$

$$2\pi r^3 = r_1^{\,3}$$

$$\sqrt[3]{2} \bullet r = r_1$$

4. *When the dough doubles in volume, by what factor does the after-rising radius increase?*

The after-rising radius increases by a factor of $\sqrt[3]{2}$.

5. *Complete the following table.*

When a Ball of Dough Doubles in Volume

before-rising radius r *in centimeters*	*after-rising radius* r_1 *in centimeters*
4	$\sqrt[3]{2} \bullet 4 = 5.03$
5	$\sqrt[3]{2} \bullet 5 = 6.3$
6	$\sqrt[3]{2} \bullet 6 = 7.56$
7	$\sqrt[3]{2} \bullet 7 = 8.82$

6. *Create a formula to express the after-rising radius in terms of the before-rising radius.*

One possible formula is $y = \sqrt[3]{2}\,x$.

7. *Create a graph showing r on the horizontal axis and on the vertical axis.*

The graph will be a line passing through (0,0).

8. *What is the slope of the line?*

The slope of the line is $\sqrt[3]{2}$.

9. *What does the slope mean in terms of the rising dough situation?*

The slope of the line represents that factor by which the linear measures of a ball of dough increase when its volume doubles.

THE MATHEMATICS OF LEMONADE—NOTES AND SOLUTION

Linear Functions and Dependent and Independent Variables

Notes

In this task, students are asked to analyze the amount of sugar in a recipe for lemonade in terms of the amount of lemon juice required. Students will be able to see that the amount of sugar is a function of the amount of lemon juice. Furthermore, students will be able to see that the function is a linear function.

Solution

Increasing a Recipe

The recipe for Lemonade tells us that for the juice of every 6 lemons, ½ cup of sugar is needed.

1. *Complete this table to show the number of lemons and the corresponding amounts of sugar required when the recipe is increased.*

Quantities of Lemons and Sugar Needed for Lemonade

lemons	*cups of sugar*
3	*¼*
6	*½*
9	*¾*
12	*1*

2. *We can think of the number of lemons and the number of cups of sugar as two variables.*

   Let x *represent the number of lemons.*
   Let y *represent the number of cups of sugar.*

   a. Which of these two variables does it make sense to call the independent variable*? Explain why.*

   It makes sense to call the number of lemons the independent variable, because the amount of sugar that is added depends on the number of lemons used and not vice versa.

b. *Which of these two variables does it make sense to call the* dependent variable*? Explain why.*

It makes sense to call the amount of sugar the dependent variable, because the amount of sugar that is added depends on the number of lemons used and not vice versa.

c. *Is the relationship between the number of lemons and the number of cups of sugar a* function*? Explain.*

Yes, this relationship is a function because each distinct number of lemons requires a distinct amount of sugar. Therefore, each value of x is mapped unto one and only one value of y.

3. *Write a formula to express the number of cups of sugar in the recipe in terms of the number of lemons.*

This recipe requires 12 lemons for every 1 cup of sugar. Thus 1 lemon requires $\frac{1}{12}$ of a cup of sugar. This is shown by the following formula: $y = \frac{1}{12}x$.

4. *Draw a graph to show the relationship between the number of lemons and the number of cups of sugar required when the recipe is increased. Record the number of lemons along the* x-*axis and the number of cups of sugar along the* y-*axis.*

The graph will be a line passing through (0,0).

5. *What is the ratio of the number of cups of sugar to the number of lemons?*

The ratio of the number of cups of sugar to the number of lemons is $\frac{1}{12}$.

6. *What is the slope of the line?*

The slope of the line is $\frac{1}{12}$.

7. *What does the slope represent in terms of making lemonade?*

The slope represents the amount of sugar required for each lemon used in making lemonade.

8. *Does the line pass through the point (0,0)?*

Yes, the line does pass through (0,0).

9. *Does the number of cups of sugar needed to make lemonade vary directly with the number of lemons used? Explain how you know.*

Yes, the number of cups of sugar vary directly with the number of lemons used because the graph is a line that passes through (0,0).

THE MATHEMATICS OF THE RAMADA—NOTES AND SOLUTION

Notes

This task provides a hands-on opportunity for students to problem-solve with concepts of scale. Students work creatively and collaboratively with measurement data to build a scale model of the Ramada, a shade structure and gathering place in the Edible Schoolyard. They can use the data given in the task or, if possible, arrange a visit the Edible Schoolyard to make their own measurements. They might use pipe cleaners or straws to construct this scale model.

Solution

Measuring the Ramada

The solution to this task will be a scale model. The model will need to be robust enough to show the intricate ceiling structure of this Ramada.

Notes

This task is designed to engage students in simple polynomial arithmetic. The dimensions of a rectangular garden bed are given as simple polynomials and students must add, subtract, multiply, or divide these in order to find the area of a strawberry patch enclosed within the bed.

Begin by asking students to work on this task individually for two minutes. Then have them work in groups of three for five minutes. Call the groups together and ask them to make suggestions as to how to get started. When a student provides what seems to be a viable starting point, have classmates discuss what makes this suggestion a good one. Record these ideas and rationale on the board or chart paper along with corresponding students' names. If a student makes a suggestion that you cannot readily understand or that may not be a viable starting point, ask for clarification. Once students have at least one viable starting strategy, have them continue working on the problem with their small groups.

As groups work, walk around the room mini-conferencing. Do not suggest or explain how to do the task. Instead, ask students to explain aspects of the task to you.

When interest in the task begins to wane, meet with the whole class once more. This time, ask students to explain ideas that you found useful or interesting as you walked about the room.

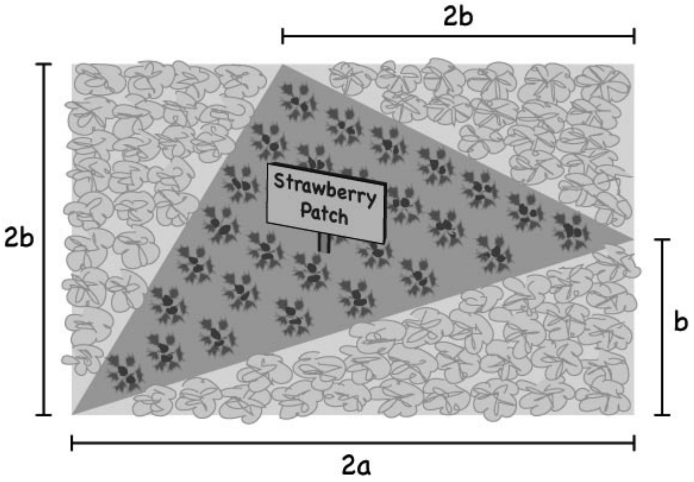
Students might suggest that they use the Pythagorean theorem to find the length of each side of the strawberry bed, for example. This is a viable starting point, but it is highly unlikely that any student will have the mathematics needed to apply the Pythagorean theorem.

Therefore, in your discussion, note strategies students may use to complete the entire task. One common approach that eighth-grade students might suggest, for example is to subtract the area of the three ground-cover right triangles from the area of the rectangular bed. This is the method that students in eighth grade could be expected to use successfully and is detailed in the solution below.

Solution

Building a Garden Bed

Students at one school are planting a strawberry patch in their school garden. They will plant the patch in a rectangular bed by partitioning the bed into 4 triangles, as shown.



Strawberries will be planted in the triangle in the middle. Ground cover will be planted in each of the 3 right triangles that border the strawberry patch.

1. *Work with classmates to create an expression for the area of the strawberry patch.*

The area of the large rectangle is $2a \bullet 2b = 2ab$.

The area of the bottom right triangle is found by multiplying half of the base (a) by the height (b). Thus, the area can be expressed as $ab : \frac{2a}{2} \bullet 2 = ab$.

The area of the upper right triangle is found by multiplying half of the base (b) by the height (b). Thus, the area can be expressed as $b^2 : \frac{2b}{2} \bullet b = b^2$.

The area of the upper left triangle is found by multiplying half of the base (b) by the height ($2a - 2b$). Thus, the area can be expressed as $2ab - 2b^2 : \frac{2b}{2} \bullet (2a - 2b) = 2ab - 2b^2$.

The sum of the areas of the 3 right triangles is as follows: $ab + b^2 + 2ab - 2b^2 = 3ab - b^2$.

The sum of the areas of the 3 right triangles subtracted from the area of the large rectangle gives the area of the strawberry patch: $4ab - (3ab - b^2) = ab + b^2$.

Therefore, the area of the strawberry patch is $ab + b^2$.

A WORD OF GRATITUDE

Ann Shannon developed these materials with support from staff at the Edible Schoolyard and teachers from Martin Luther King Jr. Middle School. Special thanks to Esther Cook for the delicious recipes and Susie Walsh Daloz for the garden lessons included here. Our thanks also to the Educational Foundation of America and Wendy Ettinger for funding the development of these materials.

So many individuals and other foundations have supported our work at the Edible Schoolyard over the last year, including Newman's Own Foundation, the Compton Foundation, the Lattner Foundation, the Zimmerman Foundation, the Charles and Helen Schwab Foundation, the Krehbil Family Foundation, the Martin Family Foundation, and Mark and Susie Buell.

About the Chez Panisse Foundation

Founded by Alice Waters in 1996, the Chez Panisse Foundation develops and supports educational programs that use food traditions to teach, nurture, and empower young people. The Foundation envisions a curriculum, integrated with the school lunch service, in which growing, cooking, and sharing food at the table gives students the knowledge and values to build a humane and sustainable future.

The Edible Schoolyard is a thriving one-acre garden and kitchen classroom for all 950 students at Martin Luther King Jr. Middle School. Through the Edible Schoolyard, students experience all aspects of growing, cooking, and sharing food at the table. Garden classes introduce the origins of food, plant life cycles, community values, and the pleasures of work, while kitchen classes allow students to prepare and eat delicious, nutritious, seasonal dishes made from produce they have grown in the garden. The Edible Schoolyard is a program of the Chez Panisse Foundation.

For more information about our work and other publications, please visit our website at www.chezpanissefoundation.org.

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